

Number of pages: 1

~~ψ_λ is not increasing, as it will only remove pixels and never add them~~

0.5 ψ_λ is increasing, because $X \subseteq Y \Rightarrow \psi_\lambda(X) \subseteq \psi_\lambda(Y)$ holds. Namely, if a connected component is in $\psi_\lambda(X)$, then apparently it was in X also. As $X \subseteq Y$, every pixel in this connected component is also part of a component in Y that is at least as big, and is therefore also in $\psi_\lambda(Y)$.

0.5 ψ_λ is anti-extensive, as $\psi_\lambda(X) \subseteq X$. This is because ψ_λ can only remove pixels, not add them.

0.5 ψ_λ is not an opening, because then we could find a structuring element A such that $\psi_\lambda(X) = X \circ A$. Suppose $X = A$, then $\psi_\lambda(A) = \bigcup_{B \subseteq A} B \circ A = A$, so λ is at least the area of the smallest connected component of A . Now move one pixel in this smallest connected component to another location, while keeping it attached to the component; call the result B . Obviously $\psi_\lambda(B) = B \circ A = \emptyset$. But this cannot be, since the ^{area} of the connected component has remained the same, so it should still pass the filter.
① correct given this def of an opening, a wider definition is that it is an idempotent, anti-extensive and increasing operator

0.5 ψ_λ is not increasing. Call the open 3×3 square A ($A = \begin{bmatrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \end{bmatrix}$) and the closed one B ($B = \begin{bmatrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \end{bmatrix}$), then $A \subseteq B$. Take $\lambda = 17$. ψ_λ will remove B as it has a perimeter of 16 pixels, but leaves A with a perimeter of 17 pixels in place; so $\psi_\lambda(B) = \emptyset$ whereas $\psi_\lambda(A) = A$, so $\psi_\lambda(A) \not\subseteq \psi_\lambda(B)$.

0.75 ψ_λ c: the snake expands to the image edges as it starts outside the bacterium
d: the snake expands to enclose only the left half as it will not pass the boundary
e: the snake will expand to the image edges at the top right, while still only enclosing the right half of the bacterium

0.5

8

the bacterium
 d: the snake will contract to a point as there is nothing for it to enclose
 e: the snake will contract to contain only the right part of the bacterium, as it will never grow to include the left part

0.75

8

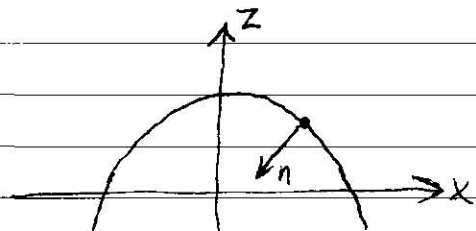
c The smoothness must be relaxed near the dimples in the center in order for the snake to fill them.

2.0

3a $\frac{\partial z}{\partial x} = -2x, \frac{\partial z}{\partial y} = -2y$
 $n = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1 \right) = (-2x, -2y, -1)$

$s = (a, b, c)$

$E(x, y) = \frac{n \cdot s}{\|n\| \|s\|} = \frac{-2ax - 2by - c}{\sqrt{4x^2 + 4y^2 + 1}}$



1.5

8

I do not see ~~the~~ ^{how} the minus signs and the square root should disappear... \rightarrow correct function of signs

b $s = (1, 0, 0) \Rightarrow a=1, b=0, c=0$

$E(x, y) = \frac{2x}{4(x^2+y^2)+1} < 0$ for $x < 0$.

1.0

In these cases, the surface is lighted from the back as the dot product is negative so the light makes an angle of over 90° with the front surface normal.

2.5

4a Say a is the ^{unit} direction vector of AB and DC, and b is the unit direction vector of AD and BC.

Then $a \cdot b = \cos \alpha$. We also have:

$(a_1, a_2, a_3) = \frac{1}{\sqrt{u_0^2 + v_0^2 + f^2}} (u_0, v_0, f) = \frac{1}{\sqrt{f^2 + 5}} (2, 1, f)$

$(b_1, b_2, b_3) = \frac{1}{\sqrt{u_0'^2 + v_0'^2 + f^2}} (u_0', v_0', f) = \frac{1}{\sqrt{f^2 + 5}} (2, -1, f)$

So $\left(\frac{1}{\sqrt{f^2 + 5}} (2, 1, f) \right) \cdot \left(\frac{1}{\sqrt{f^2 + 5}} (2, -1, f) \right) = \cos \alpha$

$\cos \alpha = \frac{1}{f^2 + 5} (4 - 1 + f^2) = \frac{f^2 + 3}{f^2 + 5}$

$f^2 + 3 = \cos \alpha (f^2 + 5) \Rightarrow f^2 = \cos \alpha f^2 + 5 \cos \alpha - 3$

$(1 - \cos \alpha) f^2 = 5 \cos \alpha - 3$

$f = \sqrt{\frac{5 \cos \alpha - 3}{1 - \cos \alpha}}$

b This would ~~require~~ ^{require} $f = \sqrt{\frac{-3}{1}} = \sqrt{-3} \notin \mathbb{R}$, which is clearly impossible.

2.0

8